Lecture six: acceleration and instantaneous acceleration In Three Dimension

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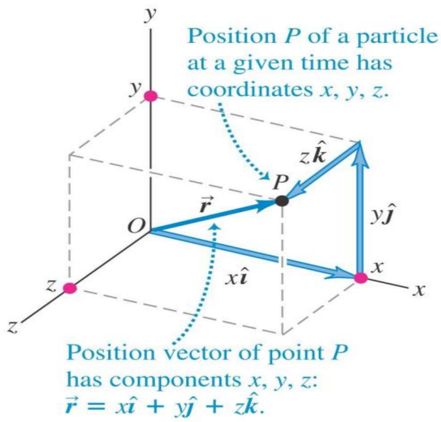
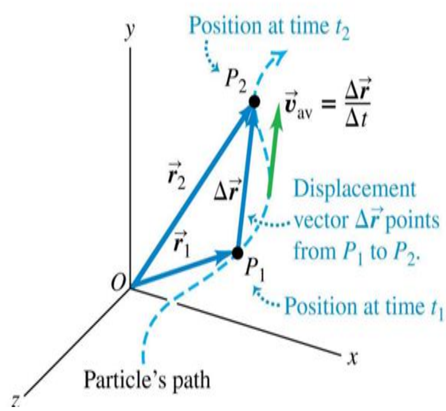
Average acceleration and instantaneous acceleration

The **position vector** $\vec{r}$ of a particle is defined as a vector whose tail is at a reference point (usually the origin O) and its tip is at the particle at point P.





Where $\hat{\imath}$,$\hat{\jmath}$, and $\hat{k}$ are unit vectors along the x, y, and z axis respectively.

For a particle that **<u>changes postion</u>** vector from $\vec{r}_1$ to $\vec{r}_2$ we define the **displacement vector** $\Delta\vec{r}$ as follows:

$$\Delta\vec{r}=\vec{r}_2-\vec{r}_1= (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$$

The **velocity vector** $\vec{v}$ at time t is definde as:

average velocity = displacement /time interval

$$\overrightarrow{\Delta v_{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}}{\Delta t}$$

Magnitude of the velocity is

$$|\vec{v}(t)| = v(t) = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

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Direction of the velocity is

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right)$$

The **average-acceleration vector** $\vec{a}$ is the ratio of the change in the instantaneousvelocity vector to the time interval:

$$\overrightarrow{a_{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x i + \Delta v_y j + \Delta v_z k}{\Delta t}$$

We define as the instantaneous acceleration as the limit:

$$\vec{a} = lim\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x i + v_y j + v_z k) = \frac{dv_x}{dt}i + \frac{dv_y}{dt}j + \frac{dv_z}{dt}k$$

The three acceleration components are given by the equations:

$$\boxed{a_x = \frac{dv_x}{dt}} \quad \boxed{a_y = \frac{dv_y}{dt}} \quad \boxed{a_z = \frac{dv_z}{dt}} \quad \boxed{\vec{a} = \frac{d\vec{v}}{dt}}$$

Example1:

The position of an electron is given by r = 3ti − 4t^2 j + 2k (where t is in seconds and r to be in meters). (a) What is v(t) for the electron? (b) In unit–vector notation, what is v at t = 2 s? (c) What are the magnitude and direction of v just then?

Solve:

(a) The velocity vector v is the time derivative of the position vector r

$$v(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3ti - 4t^2 j + 2k) = 3i - 8tj$$

(b) At t = 2s, the value of v is

$$v(t = 2s) = 3i - (8)(2)j = 3i - 16j$$

(c) Using our answer from (b), at t = 2s the magnitude of v is

$$r = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{(3)^2 + (-16)^2 + (0)^2} = 16\,\frac{m}{s}$$

$$tan\theta = \frac{v_y}{v_x} = -5.33 \Rightarrow \theta = tan^{-1}(-5.33) = -79^0$$

Example 2:

Find the average acceleration between t=0 and t=3, for the particle which is moving in a plane and whose position is given, $v = 3ti + 3t^3j + k$

Solve

The velocity as a function of time

$v = 3ti + 3t^3j + k$

At t = 0

$v_i = 0i + 0j + k$

At t = 3

$v_f = 9i + 81j + k$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{9i + 81j + k - k}{3} = 3i + 27j$$

Example 3:

Find the instantaneous acceleration at t = 1s, for the particle which is moving in a plane and whose position is given, $v = 2ti + 5t^3j + tk$

Solve

Velocity as a function of time

$v = 2ti + 5t^3j + tk$

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The instantaneous acceleration is given by the formula,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(2ti + 5t3j + tk)$$

$$\vec{a} = 2i + 15t^2 j + k$$

At t = 1

$$\vec{a} = 2i + 15j + k$$

Example 4:

A fish swimming in a horizontal plane has a velocity v_0=(4i + 1j) m/s and position vector is r_0 = (10i–4j) m relative to a stationary rock at the shore. After the fish swims with constant acceleration for 20 s, its velocity is v = (20i − 5j) m/ s. (a) what are the components of the acceleration? (b) What is the direction of the acceleration with respect to the fixed x axis? (c) Find the values of x , y and components of the velocity at t = 25 s .

<u>Solve</u>

(a) Since we are given that the acceleration is constant

$v_x = v_{0x} + a_x t$, $v_y = v_{0y} + a_y t$

to get:

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{20-4}{20} = 0.8 \text{ m/s}^2$$

And

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{-5-1}{20} = -0.3 \text{ m/s}^2$$

And the acceleration vector of the fish is

$$a = 0.8i - 0.3j$$

(b) the direction of the acceleration a is:

$$tan\theta = \frac{a_y}{a_x} = \frac{-0.3}{0.8} = -0.375$$

$$\theta = \tan^{-1}(-0.375) = -20.6^{\circ}$$

(d) $x = x_0 + v_{0x} + \frac{1}{2} a_x t^2$

$= 10 + 4 + \frac{1}{2}(0.8)(25)^2 = 360$

and

$y = y_0 + v_{0y} + \frac{1}{2} a_y t^2$

$= -4 + 1 + \frac{1}{2}(-0.30)(25)^2 = -72.8$ m

At t = 25 s the velocity components of the fish are given by:

$v_x = v_{0x} + a_x t = 4 + (0.8)(25) = 24$ m / s

and

$v_y = v_{0y} + a_y t = 1 + (-0.3)(25) = -6.5$ m / s